Date:

Chapter: Chapter 5:6 --> The Quadratic Formula and Discriminant

Objectives: Solve quadratic equations by using the Quadratic Formula.

Use the discriminant to determine the number and types of roots.

Notes:

Pumpkin catapult is an event in which a contestant builds a catapult and launches a pumpkin at a target. The path of the pumpkin can be modeled by the quadratic function $h = -4.9t^2 + 117t + 42$ where h is the height of the pumpkin and t is the number of seconds.

To predict when the pumpkin will hit the target, you can solve the equation by putting in 0 for h.

$h = -4.9t^2 + 117t + 42$

$0 = -4.9t^2 + 117t + 42$

$\frac{-42}{-4.9} = \frac{-4.9t^2}{-4.9} + \frac{117t}{-4.9}$

$t^2 - 23.87t = 8.57$

$t^2 - 23.87t + 142.4 = 8.57 + 142.4$

$(t - 11.93)^2 = 151.01$

$t - 11.93 = \pm 12.28$

$t = 24.21$ sec, -0.36

*__Quadratic Formula__ = Method used to solve any quadratic equation.

How to derive the Quadratic Formula.....

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$ax^2 + bx + c = 0$

$\frac{a}{a} \quad \frac{b}{a} \quad \frac{c}{a} \quad \frac{0}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\frac{1}{2} \cdot \frac{b}{a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$

$\frac{b}{2a} \cdot \frac{b}{2a}$

$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

*__Discriminant__ = The number under the radical in the Quadratic Formula.

How to get the Discriminant

$b^2 - 4ac$

Three Types of Discriminants

1) Positive --> 2 Real Roots

2) Negative --> 2 Complex Roots

3) Zero --> 1 Real Root

Examples:

Ex. 1 - Solve by using the Quadratic Formula.

a) $x^2 + 6x = 16$

$x^2 + 6x - 16 = 0$ ($a = 1$, $b = 6$, $c = -16$)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-16)}}{2(1)}$ ($36 + 64$)

$x = \frac{-6 \pm \sqrt{100}}{2} \rightarrow x = \frac{-6 \pm 10}{2} \rightarrow x = 2, -8$

b) $2x^2 + 25x + 33 = 0$ ($a = 2$, $b = 25$, $c = 33$)

$x = \frac{-25 \pm \sqrt{25^2 - 4(2)(33)}}{2(2)}$

$x = \frac{-25 \pm \sqrt{361}}{4}$

$x = \frac{-25 \pm 19}{4}$

$x = -11, -\frac{3}{2}$

c) $x^2 - 16x + 64 = 0$

$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(64)}}{2(1)}$

$x = \frac{16 \pm \sqrt{256 - 256}}{2}$

$x = \frac{16}{2}$ $x = 8$

d) $x^2 + 34x = -289$

$x = \frac{-34 \pm \sqrt{34^2 - 4(1)(289)}}{2(1)}$

$x = \frac{-34 \pm 0}{2} \rightarrow x = -17$

e) $3x^2 + 5x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{13}}{6}$

f) $x^2 - 8x + 9 = 0$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{8 \pm \sqrt{28}}{2}$ ($\sqrt{4}\sqrt{7}$, $\sqrt{2}\sqrt{2}$)

$x = \frac{8 \pm 2\sqrt{7}}{2} \rightarrow x = \frac{4 \pm \sqrt{7}}{2}$

g) $3x^2 + 5x + 4 = 0$

$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(4)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{-23}}{6} \rightarrow x = \frac{-5 \pm i\sqrt{23}}{6}$

$\sqrt{-1} \cdot \sqrt{23}$

i

h) $x^2 - 4x = -13$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$

$x = \frac{4 \pm \sqrt{-36}}{2}$

$x = \frac{4 \pm 6i}{2} \rightarrow x = 2 \pm 3i$

Ex. 2 - Find the value of the discriminant then state how many and what type of root.

a) $7x^2 - 11x + 5 = 0$

$(-11)^2 - 4(7)(5)$

-19

2 complex roots

b) $-7x + 15x^2 = 4$ ($a = 15$, $b = -7$, $c = -4$)

$b^2 - 4ac$

$(-7)^2 - 4(15)(-4)$

289 2 IR roots

c) $x^2 + 22x + 121 = 0$

$22^2 - 4(1)(121)$

$484 - 484 = 0$

1 IR root

Homework:

p. 298 (#14-18 Evens, 22-32 Evens, 43, 50-52)